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$$x=\sqrt{a+\frac{b}{x}}\ldots(2).$$

It is evident that if we assume for g an approximate value less than the true one, and substitute it in the right hand member of equation (2), the resulting value will be greater than the true value; if we assume for g a value greater than the true one, the resulting value will be less than the true value.

For a first approximation to the value of g, assume any convenient positive value between the limits $\sqrt{a}$ and $\sqrt{(4a/3)}$. For a second approximation, substitute the first approximation in the right hand member of equation (2) and compute the resulting value; etc.

This will give a series of approximations, alternately less and greater than the true value, towards which they converge as a limit.

Example. $x^3-6x-2=0$.

$$\sqrt{6}=2.449,$$

$$\sqrt{6+\frac{2}{2.449}}=2.610,$$

$$\sqrt{6+\frac{2}{2.610}}=2.601,$$

$$\sqrt{6+\frac{2}{2.601}}=2.6019,$$

$$\sqrt{6+\frac{2}{2.6017}}=2.601677,$$

the last being correct to five decimals.

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ON THE FORMULA FOR THE AREA OF A CURVE IN POLAR CO-ORDINATES.

By JACOB WESTLUND, Purdue University.

In deriving the formula $A=\frac{1}{2}\int_{\theta_1}^{\theta_2}\rho^2 d\theta$ for the area between a curve and two radii vectores it is customary to consider the area as the limit of the sum of infinitesimal circular sectors. This formula* may, however, be derived directly from the formula $A=\int_{x_1}^{x_2}ydx$ for the area between a curve, the axis of x, and two ordinates. To do this we assume that the curve may be divided into a finite number of parts such that for any part the abscissa x either increases or decreases constantly when we go from one extremity of the arc to the other. Let us then consider an arc AB, where x and ρ increase when θ increases from θ_1 to θ_2. Let OP be the initial line, A', B' the feet of the perpendiculars on OP from A and B, respectively, and let $OA=\rho_1$, $OB=\rho_2$.

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*It is very probable that this desirable method of proof occurs in the literature; it has been in use in Professor E. H. Moore's course in Calculus. Ed. D.

Now we have

$$OAB=OBB'-OAA'-AA'B'B=\tfrac{1}{2}\rho_2^2\sin\theta_2\cos\theta_2-\tfrac{1}{2}\rho_1^2\sin\theta_1\cos\theta_1-\int_{x_1}^{x_2}y\,dx.$$

Hence

$$OAB=\tfrac{1}{2}\rho_2^2\sin\theta_2\cos\theta_2-\tfrac{1}{2}\rho_1^2\sin\theta_1\cos\theta_1-\int_{\theta_1}^{\theta_2}\rho\sin\theta\left(\cos\theta\frac{d\rho}{d\theta}-\rho\sin\theta\right)d\theta$$

since $x=\rho\cos\theta$ increases continuously in the interval (θ_1, θ_2) and has a continuous derivative.

But by integration by parts we have

$$\int_{\theta_1}^{\theta_2}\rho^2\,\frac{d(\sin\theta\cos\theta)}{d\theta}d\theta=\rho_2^2\sin\theta_2\cos\theta_2-\rho_1^2\sin\theta_1\cos\theta_1-2\int_{\theta_1}^{\theta_2}\rho\sin\theta\cos\theta\frac{d\rho}{d\theta}d\theta.$$

Hence

$$\begin{aligned}OAB&=\tfrac{1}{2}\int_{\theta_1}^{\theta_2}\rho^2\,\frac{d(\sin\theta\cos\theta)}{d\theta}d\theta+\int_{\theta_1}^{\theta_2}\rho^2\sin^2\theta\,d\theta\\&=\tfrac{1}{2}\int_{\theta_1}^{\theta_2}\rho^2(\cos^2\theta-\sin^2\theta)d\theta+\int_{\theta_1}^{\theta_2}\rho^2\sin^2\theta\,d\theta\\&=\tfrac{1}{2}\int_{\theta_1}^{\theta_2}\rho^2d\theta.\end{aligned}$$

In exactly the same way we prove the formula for the case when x decreases through the interval (θ_1, θ_2). If we now consider any curve which may be divided into a finite number of parts, each part having the property given above, it is easily seen that the area included between the curve and two radii vectores is given by the formula, Area$=\tfrac{1}{2}\int_{\theta_1}^{\theta_2}\rho^2d\theta$.